TEACHING CHILDREN HOW TO MASTER BASIC MATH FACTS

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Definition of Terms

Basic facts for addition refer to those combinations where both addends are less than 10. Subtraction facts correspond to addition facts e.g. for the fact 3+5=8, two subtraction facts are derived, 8-3=5, and 8-5=3. Since the sum of two single digit addends can exceed 10, a subtraction fact may include a subtrahend greater than 10 e.g. 12-5=7 is a basic fact for subtraction, and this corresponds to the addition fact 5+7=12.

Basic facts for multiplication refer to those combinations where both factors are less than 10. Division facts correspond to multiplication facts e.g. for the fact 4x5=20, two division facts can be derived, 20÷5=4 and 20÷4=5.

Mastery of a basic fact means that the child can supply a quick response (less than 3 seconds) without the use of pencil and paper or other inefficient means such as counting on their fingers.

Teaching Basic Facts - General Strategies

In teaching the basic facts, children should be encouraged to use efficient reasoning strategies rather than rely on rote memorization. This is only possible if a carefully planned programme is implemented, in which clusters of related facts are identified and each cluster is taught as a group of facts with common relationships.

The traditional approach to teaching basic facts is to present each fact as a separate bit of knowledge to be mastered. For example, in teaching the concept of a number, all the combinations of the number are covered and this trend continues for each number up to 20. This approach is inefficient and may lead to rote memorisation of facts. Children may be able to recall the facts in the short term, but they may not be able to store so much information in their long-term memory.

In order achieve mastery of basic facts and retrieve facts from their long-term memory, children need to make connections between related facts. If a child knows that 2+5=7 then there is no need to teach that 12+5=17, since this can be easily derived. The approach should therefore give students general strategies and this is achieved by teaching **basic fact clusters**. These strategies focus on selecting a cluster of related facts and helping children to develop their own

strategies. Through discussion, they share their strategies with others, and seek to improve on their own. A programme of Basic Fact Mastery should draw on pupils' informal knowledge of mathematics without the teacher having to "show them how to do it".

The National Council of Teachers of mathematics emphasizes that mathematics should be taught so as to reflect the goals of *Communication, Reasoning, Problem Solving* and making *Connections* to real life situations. The approach to the teaching of Basic Facts must therefore support these goals. The following are some teaching ideas that can be used in implementing a programme on Basic Facts.

1. Use real life contexts to encourage children to appreciate the value of learning these skills and their usefulness to everyday experiences. Games and puzzles can be incorporated to motivate children.
2. Use of informal reasoning to develop an efficient strategy, e.g. a child may reason that 8+7 can be figured out by using a well-known fact such as 7+7, and adding one more. Another child may see that a group of ten can be made up by adding 8 to 2, then add the remaining 5.
3. Use opportunities to share ideas through explaining their strategies, listening to others and discussing alternative approaches.
4. Use problem solving approaches to develop strategies, such as searching for patterns, testing and verifying them, so that basic facts are grounded in strong conceptual knowledge, which can be readily retrieved.
5. Use concrete models, place value mats or number charts to assist the child in visualising the processes, especially in the early stages.
6. Use practice to reinforce strategies and consolidate ideas. Practice Worksheets should be developed for each skill.
7. Use timed tests at intervals to check on mastery, but this must only be done when children are ready. If introduced too early, it may encourage rote learning. Diagnostic tests can help to pinpoint areas of weaknesses, while inventories can serve a useful purpose in communicating to students or parents what facts the child can master at a particular time.

Previous Knowledge for Teaching Basic Mathematics Facts

For Addition and Subtraction Facts, they should be able to

- Count orally at least up to 20.
- Recognise and name the numbers from 0 to 20.
- Name the number that comes before and after a given number up to 10.
- Count backward from 10.
- Count out a set of 20 objects correctly.
- Demonstrate knowledge of addition by adding groups of items together and arriving at the correct sum.
- Demonstrate knowledge of subtraction by subtracting items from a set and arriving at the correct answer.
- Represent addition/subtraction facts using symbols.

For Multiplication Facts, they should be able to do all of the above and

- Recognise and name any number from 0 to 100.
- Count forwards and backwards by twos up to 10 and forwards by fives at least up to 50.
- Demonstrate knowledge of multiplication facts by grouping objects together and obtaining the total by counting.
- Represent multiplication facts using symbols.
- Build Multiplication Tables using concrete materials.

A note on Division Facts

The close relationship between division and multiplication makes the learning of division facts easy, once the multiplication facts are understood. For example, in working out 24 ÷ 4, one readily thinks of "What number multiplied by 4 is 24?" Hence the approach to mastering division facts is to use the corresponding multiplication fact and make the connection between both operations.

Basic Facts for each level

In presenting the facts, suggestions for each level are made, but the special aptitudes of pupils in each class must be considered before implementing a programme. Teachers must first establish what facts each member of the class knows and design the programme to meet specific needs. Many pupils reach mastery without much teaching or practice. They will have to move on to a higher skill, or problem-solving tasks can be assigned involving the skill. So teaching may often take place in small groups. Specific strategies will be suggested after the facts have been identified for each level. Revision of the facts done at previous level should also be incorporated.

In Figure 1, shown below, all the Addition Facts are displayed on the left and the Multiplication Facts on the right. In Figures 2-14, the shaded cells identify specific fact-clusters. The simple facts should be introduced early and the others later on. Before pupils enter Standard 4, all the facts should be mastered. At the upper level there will be need for review and reinforcement if pupils have not achieved mastery. Although there are 100 facts in the table, each fact (except for the double facts and square facts) is presented twice because of the Commutative Property. This property should be dealt with from time to time so that these are not seen as different facts.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 1

ADDITION AND SUBTRACTION FACTS - INFANTS-STANDARD 1

FIRST YEAR INFANTS

Addition:

- one-more-than facts, e.g. 1+1, 2+1, etc. (Figure 2).

SECOND YEAR INFANTS

Addition:

- add zero facts, e.g. 1+0, 2+0, etc. (Figure 3).
- double-facts up to 5+5 (Figure 4).
- five-facts, e.g. 1+4, 2+3, etc. (Figure 5).

Subtraction:

- one less than facts, e.g. 7-1, 6-1, etc.
- subtract-zero facts, 7-0, 6-0, etc.
- facts with zero as difference, e.g. 5-5, 4-4, etc.
- subtraction within 5, e.g. 5-1, 5-2, etc.

STANDARD ONE

Addition:

- double-facts up to 9+9 (Figure 4)
- two-more-than facts, e.g. 5+2, 3+2, etc. (Figure 6)
- ten-facts, e.g. 3+7, 6+4, etc. (Figure 5).

Subtraction:

- two-less-than facts, e.g. 10-2, 6-2, etc.
- subtraction within 10, e.g. 10-1, 10-2, etc.
- subtracting two consecutive numbers, e.g. 9-8, 6-5, etc.
- facts with two as difference, e.g. 9-7, 8-6, etc.

ADDITION AND SUBTRACTION FACTS - STANDARDS TWO & THREE

In Standard two, all the facts done previously should be reviewed. The remaining facts require some level of analysis and although many Standard 2 children are capable of explaining their strategies, there is reliance on concrete materials and practice exercises before mental skills can emerge. The Standard two child can attempt any of the following facts provided that concrete experiences are given. They should also be given opportunities to explain their strategies, but complete mastery (through mental computation) of all the remaining facts may not be achieved until they reach Standard three.

Addition: *Facts with sums beyond 10 (Figure 7) such as*

- near-double facts, e.g. 6+7, 8+9, etc.
- facts in which the addends differ by 2 so that the sum is double the number in the middle e.g. 6+8 = 7+7 = 14
- facts in which one addend is near 10, so that ten can be made up e.g. 8+5 = (8+2)+3 = 13.
- connecting addition facts with related facts, e.g. connecting 5+3 with 50+30 and 500+300, connecting 7+6 with 17+6, 27+6 and 157+6.

Subtraction: These facts are more difficult when the subtrahend exceeds 10. An approach, which is likely to be successful, is one in which strategies are identified which can work for different sets of facts. These strategies depend on mastery of addition facts and can only be successful if these facts are in place. Some of the strategies are as follows:

- identifying double and near-double subtraction facts (e.g. 12-6, 13-6) and using a *think addition* approach. Thus since 6+6 = 12, 12-6 = 6, and since 6+7 = 13, 13-7 = 6.
- using a *build-up-through-ten* strategy. In computing 15-9, start with 9 and build up to ten by adding 1, and then continue to 15 by adding 5 more.
- using a *back-down-through-ten* strategy. In computing 15-6, start with 15 and take away 5, then take one more from the ten that remained.
- connecting subtraction facts with related facts, e.g. connect 7-3 with 70-30 and 700-300 and connect 15-6 with 25-6, 35-6 and 745-6.

ADDITION FACT CLUSTERS

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 2

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 3

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 4

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 5

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 6

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 7

MULTIPLICATION FACTS - STANDARDS 1-3

Standard I

- multiplication by two (Figure 8)
- multiplication by five (Figure 9)

The above facts are directly related to addition facts and can be taught through the use of skip-counting to reinforce the concept of multiplication as repeated addition.

Standard 2

- multiplication by one and zero (Figure 10)
- multiplication by three and six (Figure 11)
- multiplication by four and eight (Figure 12)

Pupils should be able to differentiate between *multiplying by one* and *multiplying by zero*.

They should also note the difference between *adding one* and *multiplying by o*ne.

Similarly *adding zero* must not be confused with *multiplying by zero*.

Multiplying by three can be seen as double and add one more.

Multiplying by four can be seen as double and double again.

Multiplying by six can be seen as "multiply by three and double", similarly multiplying by eight can be seen as "multiply by four (double twice) then double".

Standard Three

- multiplication by 9 (Figure 14)
- multiplication by 7 (Figure 13)
- square facts (Figure 15)

The table of nines includes some patterns, which are readily seen by pupils. Since the sum of the digits in each product is always nine, it is easy to predict the result.

There is no such pattern for the table of seven, but all the facts would have been encountered before. Hence the strategy that is useful is to rely on a previously learnt fact,

e.g. 2x7 is double 7, 3x7 is 2x7 + 7, 6x7 is 5x7 +7.

Although each square fact was encountered in previous tables, these can be done again as a cluster to reinforced and linked to division facts.

Other division facts can be covered by relating to the corresponding multiplication facts, e.g. 63 - 9, can be derived from 9x ? = 63

MULTIPLICATION FACT CLUSTERS

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 8

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 9

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 10

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 11

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 12

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 13

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 14

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 15

MENTAL COMPUTATION SKILLS

The following **mental computation** skills should be done in Standard 4 or 5. These are not considered as basic facts; however, some of them can be derived from basic facts.

- Multiplying whole numbers by 10, 20, 30, etc.
- Multiplying whole numbers by 100, 200, 300, etc.
- Dividing whole numbers by 10, 100, 200 etc.
- Estimating the result of a computation (in whole numbers) to the nearest ten or hundred.
- Estimating the result of a computation (in fractions) to the nearest whole number.
- Estimating the result of a computation (in decimals or money) to the nearest whole number.
- Rounding a whole number to the nearest ten or hundred.
- Rounding a decimal number to the nearest tenth or hundredth.
- Multiplication of two and three digit numbers by 5 and 50, e.g. 32x5 = (32x10) - 2.

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